W4L5 - INVERSE LAPLACE TRANSFORM

Theorem: Suppose that f and g are continuous functions and that $\mathcal{L}[F(s)] = \mathcal{L}[g(s)]$ for $s > a$. Then $F(t) = g(t)$ for all $t > 0$. (So a function is uniquely determined by its Laplace Transform!)

Def: If f is a continuous function of exponential order and $\mathcal{L}[f(s)] = F(s)$, then we call f the inverse Laplace transform of F, and write:

$$f = \mathcal{L}^{-1}(F)$$

So ...

$$F = \mathcal{L}(f) \iff f = \mathcal{L}^{-1}(F)$$

Find the inverse Laplace transform of:

$$F(s) = \frac{1}{s-3} - \frac{16}{s^2+9}$$

$$\begin{aligned}
\mathcal{L}^{-1}[F(s)] &= \mathcal{L}^{-1}\left[\frac{1}{s-3}\right] - \mathcal{L}^{-1}\left[\frac{16}{s^2+9}\right] \\
&= e^{3t} - \mathcal{L}^{-1}\left[\frac{3 \cdot 16/3}{s^2+3^2}\right] \\
&= e^{3t} - \frac{16}{3}\mathcal{L}^{-1}\left[\frac{3}{s^2+3^2}\right] \\
&= e^{3t} - \frac{16}{3}\sin(3t)
\end{aligned}$$